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Premium Earning Patterns for Multi-Year Policies with Aggregate Deductibles

Thomas Struppeck*

Abstract[†]

Multi-year policies with large aggregate deductibles or multiple triggers raise some interesting issues about the correct amount of unearned premium reserve that a company should carry. Examples in this paper illustrate some of the difficulties that arise when trying to establish such reserves. The basic approach taken here is that the pure premium portion of the unearned premium reserve should always be adequate to cover the remaining risk. This approach, however, can lead to some unusual and controversial earning patterns; there are even situations where a negative premium is earned. In addition, the earning pattern for a particular loss scenario can differ materially from the earning pattern that is expected when the contract is written.

Key words and phrases: *unearned premium, unearned premium reserve, premium deficiency reserves, required pure premium reserve*

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1 Introduction

Statutory accounting requires that reserves be established for covered losses that have occurred but are unpaid (loss reserves) and effectively for losses that have not yet occurred, but will be covered by policies already on the books (unearned premium reserves). Furthermore, these reserves need to be separate.

A problem can arise, however, when a multi-year contract has a large aggregate deductible. If losses depleting the deductible occur faster than expected, the premium reserve at some point may be inadequate. Of course, it is also possible that those losses occur more slowly than anticipated, in which case the premium reserve may be redundant.

To deal with this potential problem with multi-year contracts, we recommend that at each point in time (or at the end of each accounting period) the pure premium portion of the unearned premium reserve should be adequate. This, in turn, implies a certain earning pattern for the premium that, in some cases, requires that a negative premium be earned.

The problems associated with the adequacy of the pure premium reserve were captured in the spirit of a hypothetical question put forward by Ruy Cardoso on CASNET in 1999. Mr. Cardoso's question is paraphrased here:

Losses are certain at \$10 per month. You cover \$20 excess \$100 in aggregate. The contract begins 7/1/xx. What is the loss reserve at 12/31/xx (ignore investment income)?

Most of this paper illustrates, using numerical examples, some of the consequences of taking the "adequate pure premium reserve" approach to establishing the unearned premium reserve (UEPR). These examples are designed to illustrate how the experience early in a multi-year excess contract affects the expected losses (to the contract, not ground-up)¹ that occurs later in the contract and how this, in turn, should affect premium reserving and earning patterns. While the examples could be made more realistic, such realism could introduce complications not relevant to the central issue. For example, in our simplification of Ruy Cardoso's question above, we assume that there are certain losses of \$20 per month. If the losses are certain, there are questions of risk-transfer.

Similarly, in Section 4, the single premium policy has an indefinite term—even though such a policy would be highly unusual. Despite

¹The losses "ground-up" refer to the losses from first dollar. The losses to the contract are those losses (limited by the limit) that are above the attachment point.

the simplifications, the examples and the technical considerations they illustrate are relevant.

Section 7 provides some comments on practical considerations, including remarks relevant to the new requirement that an actuary opine on the adequacy of the unearned premium reserve under certain circumstances.

In some cases, the approach contained herein might result, for example, in earning a premium faster than some state's regulations would allow. Naturally, one should consult with qualified accounting professionals to decide how to properly record the financials of complex or difficult contracts.

2 The Unearned Premium Reserve (UEPR)

2.1 What is Unearned Premium?

According to the glossary of the 1994 property-casualty insurance accounting text published by the Insurance Accounting and Systems Association (IASA), "Unearned premium [is] the portion of the premium applicable to the unexpired period of the policy." What is the unearned premium reserve (UEPR)? Again from the glossary, "The sum of all premiums representing the unexpired portions of the policies or contracts which the insurer or reinsurer has on its books as of a certain date ... " UEPR is a liability that represents the premium for the unexpired risks on the insurer's books.

The American Academy of Actuaries' *Statement of Principles Regarding P&C Insurance Ratemaking* (1999) states that ratemaking is prospective, and that a rate is an estimate of the expected value of future costs. Also, a rate provides for all costs associated with the transfer of risk. This paper is concerned primarily with the pure premium portion of the rate—i.e., the expected loss and loss adjustment expense, not including other expenses.

Combining these two concepts, we see that UEPR consists of the pure premiums and the other expenses for the unexpired portion of the risks that are currently on the insurer's books. From one valuation date to another, the amount of unexpired risk on an insurer's books changes: new risks may be written, and the unexpired portion of those risks that were on the books at the beginning of the period generally decreases. This is captured in the familiar accounting identity:

$$EP = WP + UEPR_{\text{begin}} - UEPR_{\text{end}} \quad (1)$$

where:

EP = The premium earned during the period;

WP = The premium written during the period;

UEPR_{begin} = UEPR at the beginning of the period; and

UEPR_{end} = UEPR at the end of the period.

Thus, other things being equal, UEPR_{end} is inversely related to the amount of premium earned. Should it happen that the UEPR_{end} for a certain policy is larger than its UEPR_{begin} without any new premium being written (we shall see below how this might happen), then equation (1) implies that the premium earned on the policy during this particular period is negative.

2.2 Example 1

We now turn to the question of the indicated UEPR for multi-year policies. For ease of exposition, let's first examine a simplified version of the problem. We will assume

- there are no reporting lags and that losses are paid as they are incurred;
- there is a maximum of one loss in each year, each loss is exactly \$1,000;
- there is no investment income; and
- the probability that a loss occurs in any given year is 10 percent and that different years are independent.

For this simplified set of assumptions, we want to compute the pure premium for the k^{th} loss during the next n years; we will denote this pure premium $PP(k, n)$. Let Policy (k, n) denote a policy covering the k^{th} loss.

To illustrate:

- $PP(1, 1)$ is the pure premium for a policy that pays \$1,000 if there is at least² one loss during year one, so $PP(1, 1) = \$1,000 \times 0.1 = \100 .

²In this first example, there can be only one loss per year so for the first year "at least one" implies "exactly one."

- $PP(1, 2)$ is the pure premium for a policy that pays \$1,000 if there is a loss during year one or year two (as we discount flows at 0 percent it does not matter which). The probability that there is no loss in two years is $0.9^2 = 81$ percent, so the probability of at least one loss is 19 percent and $PP(1, 2) = \$190$.
- $PP(2, 2)$ is the pure premium for a policy that pays \$1,000 if there are at least two losses during years one and two. As we are assuming at most one loss per year, this can happen only if there is exactly one loss in each of years one and two. The probability of this is $0.10 \times 0.10 = 1$ percent and the pure premium is \$10.

Suppose that you purchased both Policy (1,2) and Policy (2,2). You would have full coverage for two years. In fact, your coverage would be identical to first purchasing Policy (1,1) and then one year later purchasing a second Policy (1,1). Your pure premium for the first set of policies would be $\$190 + \$10 = \$200$. For the second your pure premium would be $\$100 + \$100 = \$200$ once more. This is no coincidence. Identical coverages must have identical pure premiums.

In a world ignoring transaction costs, risk and profit loads, and other expenses, where risk carriers are willing to cede or assume risks for their pure premiums, the following principle holds: If two sets of policies give identical coverage, they must have the same premium charge. If this were not so, a portfolio consisting of a long position (assumed risk) and a short position (ceded risk) could be assembled that has positive net (pure) premium, but no net risk. This would violate the economic principle of no risk-free arbitrage, also referred to as the *no arbitrage principle*.

2.3 The Required Pure Premium Reserve (RPPR)

The pure premium for a policy is equal to the expected losses at contract inception. As time passes, however, the pure premium for the remaining losses will change. The *required pure premium reserve* (RPPR) is the expected future losses (ignoring transactions costs and other expenses) over the remaining lifetime of the insurance contract. The required pure premium reserve at time t ($RPPR_t$) is the amount that a hypothetical risk carrier would require to assume the risk at time t , ignoring transactions costs and other expenses. $RPPR_t$ may depend on the loss experience up to time t .

At policy inception, the required premium reserve equals the pure premium for the policy. At policy termination, when no more losses can occur, the required premium reserve is zero. (Here and throughout

the paper we assume that losses are paid as they are incurred and that there is no reporting lag.) RPPR is similar to the unearned premium reserve (UEPR), but it has one important difference. UEPR contains premium elements other than pure premium (such as expense loads and risk loads). In world with no transactions costs, an exactly adequate UEPR is equal to RPPR; in the following discussion the terms are used interchangeably.

RPPR may depend on loss experience, as the following continuation of example 1 illustrates. The RPPR for Policy (1,2) at time $t = 0$ is the pure premium, which we computed above as \$190. After one year, we are in one of two states:

State	Probability	RPPR ₁
Loss	10%	No more cover remains; RPPR ₁ = 0
No Loss	90%	Remaining cover is Policy (1,1); RPPR ₁ = 100.

The decrease in RPPR during the first year is analogous to the (pure) premium earned during that period. The decrease in RPPR in the loss case is 190 and in the no-loss case is 90. The probability of the loss case is 10 percent, so the expected change in RPPR is $0.1 \times 190 + 0.9 \times 90 = 100$, which must be equal to the pure premium for a one-year cover (i.e., the coverage that you receive during the first year of Policy (1,2)). In fact, it is always true that the a priori expected value of the change in RPPR during a period is equal to the a priori expected value of the losses occurring during that period.

In the above example, expected losses are \$100 and the expected change in RPPR is also \$100. While the expected change is \$100, an actual change of \$100 is not possible in this example. (It is either \$90 or \$190.)

3 The Adequate Pure Premium Reserve Approach

Using this approach, the change in RPPR is a correct measure for pure premium earned during the period, and the pure premium portion of UEPR should be RPPR. Applying this approach to the example of the previous section: in the no-loss case, we would earn premium of \$90 during the first period. In the loss case we would earn premium of \$190.

Under current accounting rules: in the loss case, because there is no more cover, all future premiums would be accrued and earned in the current period,³ so earned (pure) premium would be \$190, just as the adequate pure premium reserve approach indicates. In the no-loss case, I believe that most companies would simply earn half of the pure premium (\$95) during the first year (and some might recognize that they have a \$5 premium deficiency, as the pure premium for year two is \$100).

My view is that at policy inception we expect to earn \$100, but that in fact we earn either \$190 or \$90 depending on our experience. This however can lead to some odd results.

Consider the expected change in RPPR for Policy (2,2) during year one. This policy pays \$1,000 for the second loss in two years. The pure premium for this policy is \$10, so this is RPPR at time 0.

After one year we are again in one of two states:

State	Probability	RPPR ₁
Loss	10%	Remaining cover is Policy (1,1); RPPR ₁ = 100
No Loss	90%	As there can be only one loss per year, there can now be no second loss: RPPR ₁ = 0.

In the no-loss case, which occurs 90 percent of the time, the decrease in RPPR is \$10. In the loss case, the decrease in RPPR for Policy (2,2) is -\$90. The expected decrease in RPPR is $0.9 \times 10 + 0.1 \times -\$90 = 0$.

The premium earning principle tells us that this must be the expected value of losses occurring during the first year. Does this make sense? Yes! This policy pays only on the second loss, and because we assume there can be only one loss per year, the second loss cannot occur during year one. That is why the expected losses during year one are zero.

3.1 Standard Premium-Accrual Methodology Considerations

I am not certain how companies would account for the above cover today. Some would argue that because the second loss cannot occur in

³Under U.S.-GAAP, at least for reinsurers, this is the content of EITF93-6, Issue 3 "How should the ceding and assuming companies account for changes in future coverage resulting from experience under the reinsurance contract?"

year one, no premium should be earned in year one on this cover; they would earn all \$10 in year two. Others might earn \$5 in the first year and \$5 in the second year.

I would argue that in the no-loss case all \$10 should be earned in the first year, but that in the loss case –\$90 should be earned in the first year. The adequate pure premium reserve approach implies that the amount of pure premium earned during a period must be that amount such that the remaining RPPR contains exactly the expected pure premium required for the remaining policy period given the losses that have occurred to date.

At inception, the company's expectation is to earn nothing during year one on this policy because the insured event could not occur during this period. But in fact one of two things happens: they have either an underwriting gain of \$10 or an underwriting loss of \$90.

The standard premium accrual procedure referred to earlier (i.e., accruing all future premium when no more cover remains) together with an application of the no arbitrage principle leads to the same conclusion as the adequate pure premium reserve approach, as we will now illustrate.

Recall that the portfolio consisting of Policy (1,2) and Policy (2,2) together give identical coverage to the portfolio consisting of Policy (1,1) along with a one year deferred Policy (1,1). By the no arbitrage principle, the premiums and how they are earned should be the same. During year one, the premium earned on Policy (1,1) is equal to 100. The premium earned during year one on each of Policy (1,2) and Policy (2,2) depends on the results of year one:

(i) The Loss Case:

Probability = 10 percent

Policy (1,2) earns a premium of \$190 implies

Policy (2,2) earns a premium of –\$90

or

(ii) The No-Loss Case:

Probability = 90 percent

Policy (2,2) earns a premium of \$10 implies

Policy(1,2) earns a premium of \$90.

In the loss case, the premium earned on Policy (1,2) is \$190 by the standard premium accrual procedure. Using the no arbitrage principle,

because the total premium earned on the two policies during year one must be \$100, the premium earned on Policy (2,2) must be $-\$90$.

Similarly, in the no-loss case, the premium earned on Policy (2,2) should be all \$10, because no coverage remains. No arbitrage forces the premium earned on Policy (1,2) to be \$90, because the sum must be \$100.

If one is uncomfortable with earning all of the premium for Policy (2,2) in the no-loss case in year one, consider what happens to the pair of policies in year two given that there is no loss in year one. The coverage is identical to the coverage afforded by a one year deferred Policy (1,1), so the earned premium in year two must be the same: \$100. The coverage during year two for Policy (1,2) is the same as for a Policy (1,1) because we are given that there is no loss in year one. The premium earned on Policy (1,2) during year two must be \$100. Because the total premium earned is also \$100, no premium can be earned on Policy (2,2). Over the life of Policy (2,2) \$10 must be earned; if none is earned in year two, all of it must be earned in year one.

3.2 Reconciling Total Earnings

The total amount of pure premium earned during the life of the policy is always equal to the initial pure premium. If some negative premium is earned during one period, it is recovered in later periods (or is balanced by some overearning in prior periods). The total change in RPPR from contract inception to contract termination is the a priori pure premium. This is an important point. The negative premium earned is not new premium, the written premium stays the same—it is just earned in a different pattern.⁴

UEPR for a given policy is amortized over the policy's term. This amortization occurs according to some amortization schedule. For most lines of business this amortization schedule is linear over the term. This linearity produces the familiar pro-rata earning pattern. This pattern is theoretically correct for a policy with no aggregate deductible, no aggregate limit, and an underlying loss process that has a compound Poisson distribution. For a further discussion of compound distributions see, for example, Bowers et al., (1997, Chapter 12). For certain lines of business (e.g., extended warranty, ocean marine cargo cover, credit insurance on a declining balance) other amortization patterns and, hence, earning patterns are used. The adequate unearned

⁴It should be noted that the process of setting the UEPR to the currently required pure premium reserve is nothing more than a mark-to-market of the outstanding UEPR.

premium reserve process described above can be thought of as adjusting this amortization schedule to include the latest data.

Traditionally, one thinks of unearned premium reserves flowing into loss reserves and surplus as the policy term progresses. Sometimes the losses occur more slowly than expected, and an unexpectedly large portion of this flow goes to surplus. Other times losses occur more rapidly than expected, and (unfortunately) in these cases surplus may flow into loss reserves. In the example above, it is the unearned premium reserve, not the loss reserve, that has become inadequate and requires supplementation from surplus.

4 More Examples

4.1 Example 2: A Less Simplified Example

This example allows for more than one loss in each year. For simplicity, we assume that in each year there are 0, 1, or 2 losses with probabilities $1/2$, $1/3$, and $1/6$, respectively. Losses are still constant but the constant loss amount will be \$216 instead of \$1,000. We continue to ignore investment income.

The pure premiums for Policy (k, n) may be computed as follows. First compute the probability of having exactly k losses by the end of year n ; the result of this calculation⁵ is displayed in Table 1. Then sum the probabilities in Table 1 to produce the probability of having at least k losses in n years; see Table 2 for these values. Finally, multiply the probabilities in Table 2 by the constant loss amount of \$216 to compute the pure premiums shown in Table 3.

Consider Policy $(2, 3)$, which covers the second loss in three years. The pure premium for this coverage is \$135. How much of this premium do we expect to earn during the first year?

Half of the time there will be no loss during the first year, and RPPR for the last two years of the policy must be \$90—the pure premium for Policy $(2, 2)$. In this case $\$135 - \$90 = \$45$ would be earned in the first year.

⁵The probabilities are most easily computed recursively. For example:

$$\Pr(2, 2) = 1/2 \times \Pr(2, 1) + 1/3 \times \Pr(1, 1) + 1/6 \times \Pr(0, 1).$$

That is, the only way to have exactly two losses at the end of year two is to have had no loss in year two and exactly two losses in year one, or exactly one loss in year two and one loss in year one, or two losses in year two and no loss in year one. (Here the events joined by “and” are independent and the events joined by “or” are mutually exclusive.)

Similarly, one-third of the time there will be one loss during the first year; then RPPR for the last two years must be \$162 (the pure premium for Policy (1,2), which is equivalent to the remaining coverage) and $\$135 - \$162 = -\$27$ would be earned during the first year.

Finally, one-sixth of the time there are two losses in year one. In this case there is no more coverage available. RPPR for the last two years is zero, and the full \$135 would be earned during year one.

Combining the above calculations for the first year earned premiums we find that *at policy inception* the expected earned premium for year one is

$$1/2 \times \$45 + 1/3 \times -\$27 + 1/6 \times \$135 = \$36.$$

Year three's expected earnings are similarly easy to calculate: during the first two years of the cover there is a $1/2 \times 1/2 = 1/4$ chance that there have been no losses and a $1/2 \times 1/3 + 1/3 \times 1/2 = 1/3$ chance of exactly one loss. From Table 2, we see that the pure premium for Policy (2,1) is 36 and for Policy (1,1) is 108. From this we see that *at policy inception* we expect to earn $1/4 \times \$36 + 1/3 \times \$108 = \$45$ during year three.

During the life of the policy we will earn exactly \$135. If *at policy inception* we expect to earn \$36 in year one and \$45 in year three, it follows that we must expect *at policy inception* to earn $\$135 - \$36 - \$45 = \54 during year two.

Does this mean that we should earn the premium over the three years in this pattern: \$36, \$54, \$45? No, because these are a priori expectations. As we have seen in earlier sections, the premium earned during year one need not equal the a priori expected earned premium. Also, at the end of year one our expectations for the earnings in years 2 and 3 will probably be different than they were at inception.

The first two rows of Table 3 contain all the information needed to compute the actual amount of premium earned to date at the end of each year. For example, suppose there is exactly one loss, and it occurs in year two. Then we should earn \$45 in the first year, because when we start year two, the remaining coverage is the second loss in two years: a Policy (2,2). During year three we are in a first-loss position, so we need to earn \$108 because at the start of year three, the remaining coverage is the first loss in one year: a Policy (1,1). Because the total amount earned over the three years must be \$135, we find that the year two (actual) earnings must be $-\$18$. So the actual earning pattern observed in this case would be (\$45, $-\$18$, \$108), which differs markedly from the a priori expectation.

Table 1
Probability of Exactly k Losses in n Years

k	$n = 1$	$n = 2$	$n = 3$
0	50.00%	25.00%	12.50%
1	33.33%	33.33%	25.00%
2	16.67%	27.78%	29.17%
3	0.00%	11.11%	20.37%
4	0.00%	2.78%	9.72%
5	0.00%	0.00%	2.78%
6	0.00%	0.00%	0.46%

Table 2
Probability of at Least k Losses in n Years

k	$n = 1$	$n = 2$	$n = 3$
0	100.00%	100.00%	100.00%
1	50.00%	75.00%	87.50%
2	16.67%	41.67%	62.50%
3	0.00%	13.89%	33.33%
4	0.00%	2.78%	12.96%
5	0.00%	0.00%	3.24%
6	0.00%	0.00%	0.46%

Table 3
Pure Premiums for Policy (k, n)

Loss k	$n = 1$	$n = 2$	$n = 3$
1	108	162	189
2	36	90	135
3	0	30	72
4	0	6	28
5	0	0	7
6	0	0	1

4.2 Example 3: An Indefinite-Term Example

In this example we will assume a $1/10$ chance of loss each year and return to the simplified model of at most one loss per year. Loss severity is assumed constant at \$3,000. We will continue to ignore investment income. The policy that we consider in this example covers one loss, but has no time limit. The policy will stay in effect until there is a loss, at which time it will pay \$3,000.⁶

4.2.1 Pure Premium and Earning Patterns

What is the pure premium for this coverage? Let P be this premium. Then P must pay for two things. One-tenth of the time there is a loss during year one of \$3,000 and $RPPR_1 = 0$. The other nine-tenths of the time, there is no loss in the first year, and $RPPR_1$ is the pure premium for a policy that pays \$3,000 whenever the loss occurs—but this is exactly what P is. We have:

$$P = 1/10 \times (\$3,000 + 0) + 9/10 \times (0 + P).$$

Solving for P , one finds $P = \$3,000$.

Upon reflection this is not surprising, as \$3,000 will be paid out eventually. (Recall that we are still ignoring investment income.) The pure premium equals the expected loss, which is \$3,000.

How does one earn the premium for such a policy? In the loss case, the premium earned in year one is \$3,000; in the no-loss case the premium earned in year one is \$0 (because $RPPR_1$ remains at \$3,000). At policy inception the expected earned premium for the first year is \$300.

What about later years? The answer depends on when you ask the question.

At the start of the first year, we expect to earn \$270 during the second year and \$243 during the third. But these are the a priori expectations at the start of the first year; after one year has passed there has been either one loss or no loss, and with this additional information the expected values for earned premium change.

At the start of the second year there are two possibilities: either there is a loss in year one (in which case no coverage remains) or there is no loss in year one (in which case there is coverage for year two). Also, because we are assuming no late reporting, you will know which case applies. The conditional expectation (given no loss in year one)

⁶This example is akin to a single premium whole life insurance policy.

for the premium earned in year two is \$300. Similarly, the conditional expectation (given no loss in year one) for the premium earned in year three is \$270. Similarly, the conditional expectation (given no loss in years one and two) for the earned premium in year three is \$300.

The expected earning pattern at the start of any year, for that and subsequent years, is (\$300, \$270, \$243, ...), with each term being 9/10 of the previous term. When a year passes without loss, each of these terms shifts forward. It should come as no surprise that this infinite geometric series sums to \$3,000.

Why is no premium earned during no-loss years? Because RPPR at the start of the no-loss year is \$3,000, and it is also \$3,000 at the end of the year. The change in RPPR, in this case \$0, is the earned premium. During a loss year, RPPR is \$3,000 at the start of the year, and it is \$0 at the end of the year (because no more coverage remains). The amount earned during the year is \$3,000.

The company shows no underwriting gain or loss, regardless of the outcome. In the no-loss case there is no movement in the reserves; in the loss case RPPR becomes the loss reserve. This is a consequence of the indefinite policy term. Because the cover continues until there is a loss, having a no-loss year only delays the inevitable payment; without investment income, the delay does not benefit us. We relax this restriction below.

4.2.2 The Impact of Investment Income

Let's take into account investment income. Assume that all losses are paid at the end of the year and that invested funds earn interest at a rate of 5 percent. The equation for the present value of the pure premium then becomes:

$$P = 1/10 \times \frac{\$3,000}{1.05} + 9/10 \times \frac{P}{1.05}.$$

One-tenth of the time we pay a loss of \$3,000 (discounted one year) and nine-tenths of the time the present value of $RPPR_1$ is P (discounted one year). Solving for P , we find that $P = \$2,000$.

How should this premium be earned? Should the fact that we now consider investment income affect how we earn the premium?

Suppose that we have a loss in year one. Then, as before, $RPPR_1 = 0$, so we earn the full \$2,000 during year one. We also have investment income of \$100. On the other hand, suppose that we have no loss in the

first year. Then $RPPR_1 = \$2,000$, and again we have investment income of \$100. What should be done with the investment income?

To investigate that question, we examine an alternative way to construct this same coverage. Consider an annual policy that pays \$1,000 at the end of the year if there is a loss, for a premium payable at the end of the year⁷ of \$100 (the pure premium for the policy). In effect, this policy provides similar coverage to the first year of the original policy, subject to a \$2,000 self-insured retention. Imagine that the insured sets aside this \$2,000 in a special account. During the year, \$100 in investment income is earned on the \$2,000 (this is paid to the insurer as premium) and, if there is a loss, the \$2,000 set aside and the \$1,000 from the insurer combine to provide the \$3,000.

With a one-time premium of \$2,000 and a limit of \$3,000, the insurer has only \$1,000 at risk. So in this second set-up, the insurer is entitled to only \$100 (= $\$1,000 \times 10\%$) in annual pure premium. This, as we have seen, is the investment income generated by the one-time premium payment of \$2,000.

We see that the insured can obtain identical coverage in two ways: by setting aside the \$2,000 and paying an annual premium of \$100 in arrears or by paying a one-time premium of \$2,000. The no arbitrage principle says that because the two coverages are identical, their pure premiums must be equal. In order for this to work, we need to view the investment income on (discounted) premium as premium—this is implicit in the pricing equation.

Now we can determine the earning pattern for the original multi-year policy and answer the question about what to do with the investment income. In a year with no loss, premium of \$100 is earned. In a loss year, premium of \$2100 (the original premium plus one year's investment income) is earned.

This result is related to the paid-up insurance formula for life reserves; see, for example, Bowers et al., (1997, Chapter 7).

4.3 Example 4: An Example with Expenses

In the real world, UEPR contains many components in addition to RPPR's pure premium. There may be, for example, on-going contract maintenance expense.⁸ Effectively, such expense forms an annuity that runs until contract termination. One quick example will give a flavor of the complications.

⁷The premium is made payable at the end of the year to remove timing effects.

⁸Had these expenses have been deferred policy acquisition expenses, there would be additional accounting complications.

Recall the earlier example of an indefinite-term policy that pays \$3,000 when the loss occurs, has annual loss probability of 10 percent, and no investment income. Assume that on-going contract maintenance expense is \$150 per year. Letting G stand for the expense-loaded premium, the premium equation now reads:

$$G = 1/10 \times (\$3,000 + \$150) + 9/10 \times (G + \$150).$$

That is, one-tenth of the time we have expenses of \$150 and a loss of \$3,000, and the other nine-tenths of the time we have expenses of \$150 and $RPPR_1 = G$ (because of the indefinite term). Solving for G , we find that $G = \$4500$.

The company with this risk on its books suffers an underwriting loss (after expenses) of \$150 each year that there is no loss, but has an underwriting gain of \$1350 the year that the loss occurs!⁹

The interested reader may find it amusing to calculate the effect on this example of including 5 percent investment income.

5 Some Practical Ramifications

Though the preceding examples illustrate some of the theoretical issues, the practicing actuary must consider the broader practical effects of any change to common practice. Questions of materiality and practicality also should be addressed.

5.1 Actuarial Reserve Opinions

The National Association of Insurance Commissioners' (NAIC) *SAO Instructions for Property-Casualty* (1998) specifies that the SCOPE paragraph include the reserve for direct, ceded, and net unearned premiums. It also specifies that these three items must be covered in the opinion and relevant comments paragraphs. This applies to all insurers that write direct and/or assumed contracts or policies (excluding financial guaranty, mortgage guaranty, and surety contracts) with terms of

⁹What's happening here is that we have an annuity with an expected life of ten years funding the expenses. When we have a no-loss year, the expected life of the annuity stays at ten (instead of decreasing to nine) and we show an underwriting loss of the difference. When we have a loss year, the expense annuity is no longer needed (its expected life drops from ten to zero). The release of the reserve supporting this annuity yields the underwriting gain.

13 months or more, which the insurer cannot cancel and for which the insurer cannot increase premiums during the term.

The insurer is required to establish an adequate unearned premium reserve. For each of the three most recent policy years, the gross unearned premium reserve must be no less than the largest result of three tests. The three tests (in slightly simplified form) are:

1. The best estimate of the amounts refundable to the contract holders at the reporting date.
2. The gross premium multiplied by the ratio of (a) over (b) where:
 - (a) Equals the projected future gross losses and expenses to be incurred during the unexpired term of the contracts; and
 - (b) Equals the projected total gross losses and expenses under the contracts.
3. The amount of the projected future gross losses and expense to be incurred during the unexpired term of the contracts (as adjusted), reduced by the present value of the future guaranteed gross premiums.

The examples in this paper are intended to be non-cancelable insurance contracts with fixed premiums. The contract terms are more than 13 months in length. The rule applies, except for the proscribed lines of business. How do our examples fare under these tests?

For simplicity, we shall assume that there are no refund provisions in the policy, so the Test 1 lower bound on the unearned premium reserve is zero.

Test 2 requires that we estimate gross losses and expense. The examples in this paper for the most part have been concerned with pure premiums (i.e., only the expected losses, with no provision for expenses). Under the simplifying assumption that expenses are zero, Test 2 tells us to estimate the projected future gross loss to be incurred and to divide this by the projected total gross loss. This ratio is then multiplied by the gross premium to obtain the second lower bound on the unearned premium reserve.

Test 3 requires that the unearned premium reserve be at least as large as the expected future losses and expenses to be incurred during the contract (as adjusted). The amount of the projected future gross losses to be incurred is exactly RPPR at the statement date. The adjustments in question are for future premiums and for investment income until the loss is incurred but not beyond. Our examples have no future

premiums and our losses are assumed to be immediately payable. [The test also specifies a company-specific maximum interest rate. We will assume that 5 percent meets this test.]

In our examples, RPPR is the lower bound on the unearned premium reserve specified by Test 3.

5.2 Perspectives on Aggregate Deductible Business

In a multi-year contract with an aggregate deductible, the experience of the first few years can influence the required premium reserve in two ways. First, the aggregate deductible may be depleted more rapidly or more slowly than planned; second, adverse or favorable experience during the initial period may influence one's view of the future ground-up experience. This paper addresses only the former.

There is an additional way to view such policies. The later years of a multi-year policy with an aggregate deductible can be thought of as excess layers, each year/layer having a retention that depends on the earlier years' experience. If the total losses to date have been small, little of the aggregate deductible has been eroded and the retention (the remaining aggregate deductible) for the later years is higher. Because higher layers have lower premiums, RPPR is small. Similarly, if early experience has been unfavorable, much of the aggregate deductible will have been eroded. The retention will be lower and RPPR will be large. In essence, early experience determines to which layers the later years' coverage corresponds.

5.3 What to Do About Negative Premium?

In chapter 14 of the IASA text, David L. Holman and Chris C. Stroup discuss U.S.-GAAP accounting for P&C insurers. Under U.S.-GAAP there is a notion of a premium deficiency reserve (PDR). Holman and Stroup write:

Projections, therefore, are periodically updated, based on new information about expected cash flows. GAAP requires that a premium deficiency be recognized if the sum of expected loss and loss adjustment expenses, expected dividends to policyholders, maintenance costs, and unamortized (or deferred) policy acquisition costs, exceed the related unearned premiums related thereto.

If there is a deficiency, the unamortized policy acquisition costs are reduced to make up the shortfall. If that alone is not sufficient, a liability

is reported for the remaining deficiency. Interestingly, Canadian statutory accounting provides a line item (Line 15) for premium deficiency (see chapter 18 of the IASA text). European actuaries speak of the reserve for unexpired risks, which is similar in concept to a combined unearned premium and premium deficiency reserve.

So, under U.S.-GAAP one might establish a PDR to handle negative premium earnings. Effectively, a negative premium is earned by the reduction of an asset (the unamortized policy acquisition cost) and/or the establishment of an additional liability.

Statutory accounting does not have the notion of a premium deficiency, although in principle one could include one by using the write-in lines. Due to U.S. income tax regulation, there may be a material difference between treating the shortfall as premium or as some other type of liability. The interested reader should see chapter 13 of the IASA text or Almagro and Ghezzi (1988).

5.4 Is It Loss or Is It Premium?

The argument can be made that instead of altering the premium earning methodology, we should establish loss reserves corresponding to the losses that are eroding the aggregate deductible. That is, there is an increase in expected losses to the cover caused by events that have occurred prior to the statement date. The amounts are not in dispute; they would be exactly the amount needed to make the booked reserve match RPPR. The difference is that these reserves would be characterized as loss instead of premium.

But these reserves behave more like premium than loss in two important ways. First, they amortize over the remaining policy period. To see the second reason, consider a two-trigger two-year policy. In order for the policy to pay, two events, A and B, must occur during a two-year period. Say event A occurs in year one, and as a result some additional reserve (either a loss reserve or a premium deficiency reserve) is needed. Suppose now you wanted to completely reinsure this risk. You could do this by purchasing cover for event B. Observe that this reinsurance is completely prospective. Being prospective, it should be funded from premium reserves, not loss reserves.¹⁰

¹⁰Claims-made policies and sunset clauses in reinsurance agreements can further blur the line between premium reserves and loss reserves. Suppose that an event has occurred, but that it has not been reported yet. Assuming that a reserve is appropriate, should it be premium or loss? This reserve amortizes over the remaining reporting period (acts like premium). On the other hand, the underlying loss event has already occurred. Is the reporting a second trigger?

6 Conclusions

We could use the adequate pure premium reserve approach to answer Mr. Cardoso's question, which was mentioned in Section 1 above: Losses are certain at \$10 per month. You cover \$20 excess \$100 in aggregate. The contract begins 7/1/xx. What is the loss reserve at 12/31/xx?

Assuming no expenses or investment income, UEPR would be \$20 (because that is RPPR remaining), and the loss reserve would be \$0 (because no covered loss has occurred). No premium (positive or negative) would have been earned to date.

The adequate pure premium reserve approach outlined in this paper is internally consistent, even though it leads to some controversial implications such as negative earned premium. But the idea of negative earned (and written) premium already is used in some instances, such as the treatment of ceded proportional reinsurance. U.S.-GAAP and Canadian accounting have a notion of a premium deficiency reserve (PDR), and in some European jurisdictions there is a notion of an unexpired risk reserve. These entries could be used to record unexpected changes in the required premium reserve.

There are some operational problems, however, with the negative premium approach: it may distort loss and expense ratios; it can make budgeting difficult; and, for U.S. taxpayers, the treatment of UEPR for U.S. taxation is different than for other reserves, which could lead to complications.

The good news is that, on average, the standard methodology should give the same results as this method for a large book of uncorrelated risks, written evenly throughout the year. The analysis outlined in this paper is probably justified for those risk carriers with a few large risks or for single risks that are large enough to distort the book.

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